Задача 4Б

Калин. А.

1) $S(\omega) = \operatorname{Re} S(\omega) + i \operatorname{Im} S(\omega)$

$$\frac{2\pi}{a} \cdot S(\omega) = \int_0^\tau \left(1-\frac{t}{\tau}\right) e^{-i\omega t}\, dt =$$

$$= \int_0^\tau e^{-i\omega t}\, dt - \frac{1}{\tau}\int_0^\tau t\, e^{-i\omega t}\, dt =$$

$$= \frac{-1}{i\omega} e^{-i\omega t}\Big|_0^\tau + \frac{1}{i\omega\tau}\int_0^\tau t \cdot d\left(e^{-i\omega t}\right) =$$

$$= -\frac{1}{i\omega}\left(e^{-i\omega\tau}\right) + \frac{1}{i\omega\tau}\left[t e^{-i\omega t}\Big|_0^\tau - \int_0^\tau e^{-i\omega t}\, dt\right] =$$

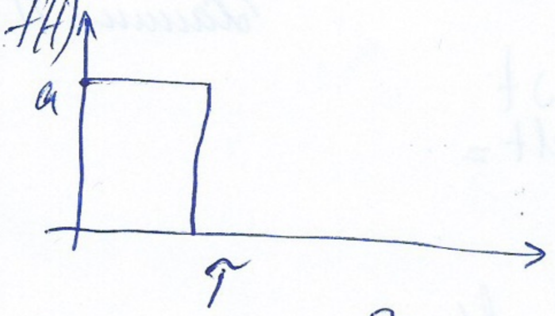
$$= \frac{e^{i\omega\tau}}{i\omega}\left(e^{\frac{i\omega\tau}{2}} - e^{-\frac{i\omega\tau}{2}}\right) + \frac{1}{i\omega\tau}\left[\tau e^{-i\omega\tau} + \frac{1}{i\omega} e^{-i\omega t}\Big|_0^\tau\right]$$

$$= \cancel{-\frac{1}{i\omega} e^{-i\omega\tau}} + \frac{1}{i\omega} + \cancel{\frac{1}{i\omega} e^{-i\omega\tau}} - \frac{1}{\omega^2\tau}\left(e^{-i\omega\tau} - 1\right)$$

$$= -\frac{1}{\omega} i + \frac{1}{\omega^2\tau} - \frac{1}{\omega^2\tau}\left(\cos\omega\tau - i\sin\omega\tau\right) =$$

$$= \underbrace{\frac{1}{\omega^2\tau}\left(1-\cos\omega\tau\right)}_{\frac{2\pi}{a}\operatorname{Re} S(\omega)} + \underbrace{i\frac{1}{\omega}\left(-1+\frac{1}{\omega}\sin\omega\tau\right)}_{\frac{2\pi}{a}\operatorname{Im} S(\omega)}$$

2) Прямоугольный импульс



$$\frac{2\pi}{a} S(\omega) = \int_0^\tau e^{-i\omega t}\, dt = -\frac{1}{i\omega} e^{-i\omega t}\Big|_0^\tau =$$

$$= -\frac{1}{i\omega}\left(e^{-i\omega\tau} - 1\right) = \frac{1}{i\omega}\left(1 - e^{-i\omega\tau}\right) =$$

$$= e^{-\frac{i\omega\tau}{2}} \frac{1}{i\omega}\left(e^{\frac{i\omega\tau}{2}} - e^{-\frac{i\omega\tau}{2}}\right) =$$

$$= \tau \cdot e^{-\frac{i\omega\tau}{2}} \left[\frac{\sin(\omega\tau/2)}{(\omega\tau/2)}\right]$$

$$S(\omega) = \frac{a\tau}{2\pi} \cdot \left|\frac{\sin\frac{\omega\tau}{2}}{\frac{\omega\tau}{2}}\right| ; \quad 2\pi\Delta f \approx \frac{3.78 \cdot 2}{\tau}$$

$\tau = 1$мс

$$\Delta f \approx \frac{1.2}{\tau} = \begin{cases} 1.2 \text{ кГц} \\ 1.2 \text{ МГц} \end{cases}$$

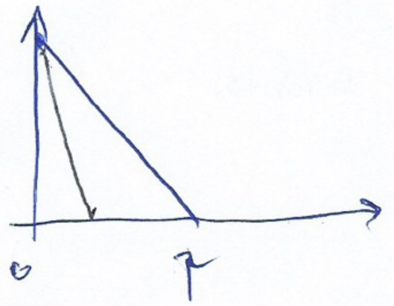
$\tau = 1$мкс.

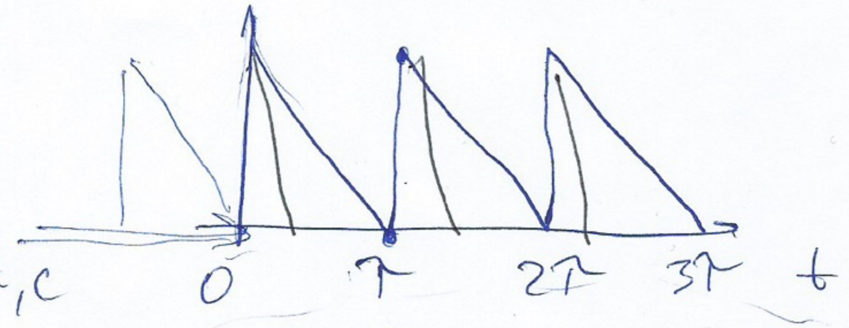
3) $\Delta(\omega\tau) \approx 10$

$$\Delta\omega \approx \frac{10}{\tau} \Rightarrow 2\pi\Delta f \approx \frac{10}{\tau} \Rightarrow \Delta f =$$

$$= \frac{1.55}{\tau} = \begin{cases} 1.55 \text{ кГц} & \tau = 1\text{мс} \\ 1.55 \text{ МГц} & \tau = 1\text{мкс}. \end{cases}$$

4)





$$S(\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} f(t)\, e^{-i\omega t}\, dt \qquad f(t) = \sum_{n=-\infty}^{+\infty} a_n\, e^{in\omega_0 t}$$

$\omega_0 = \frac{2\pi}{\tau}$;

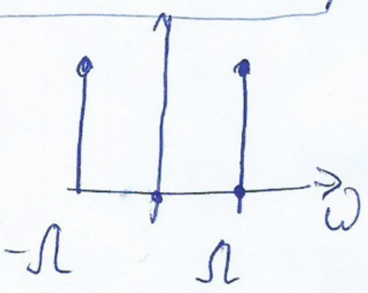
$\cos \Omega t = f(t)$

$$S(\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \cos \Omega t \cdot e^{-i\omega t}\, dt =$$

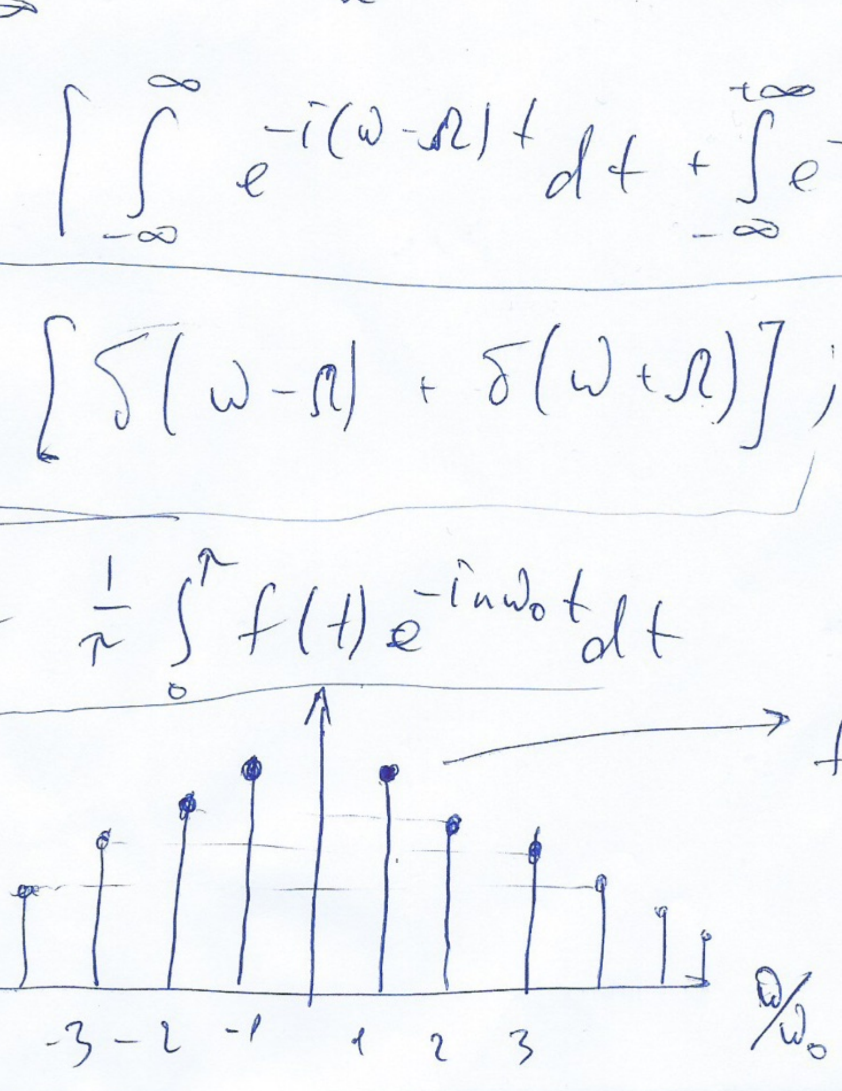
$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty} \frac{e^{i\Omega t} + e^{-i\Omega t}}{2}\, e^{-i\omega t} =$$

$$= \underbrace{\frac{1}{4\pi}\left[\int_{-\infty}^{\infty} e^{-i(\omega-\Omega)t}\, dt + \int_{-\infty}^{+\infty} e^{-i(\omega+\Omega)t}\, dt\right]}_{} =$$

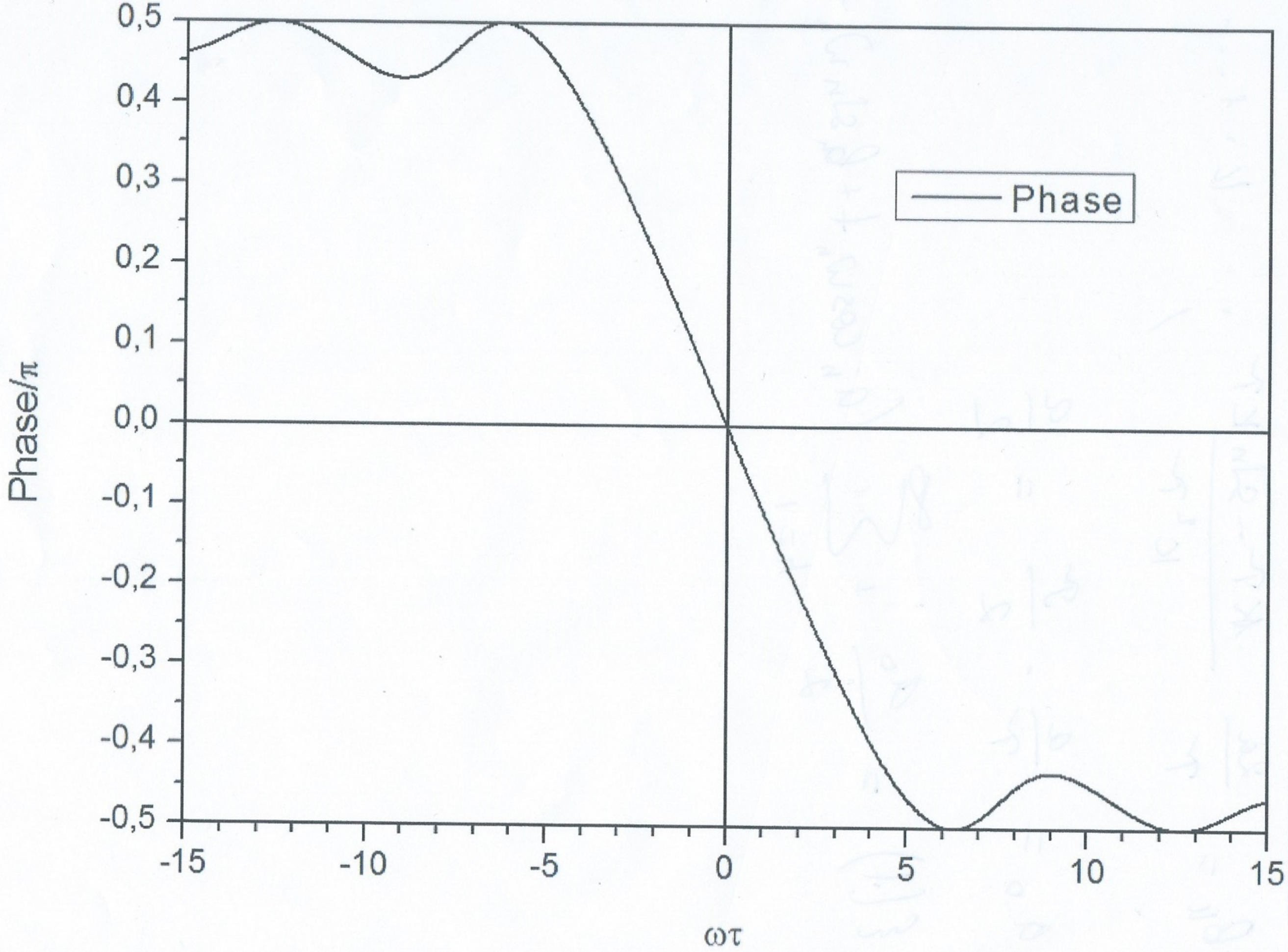
$$= \underbrace{\frac{1}{2}\left[\delta(\omega-\Omega) + \delta(\omega+\Omega)\right]}_{};$$

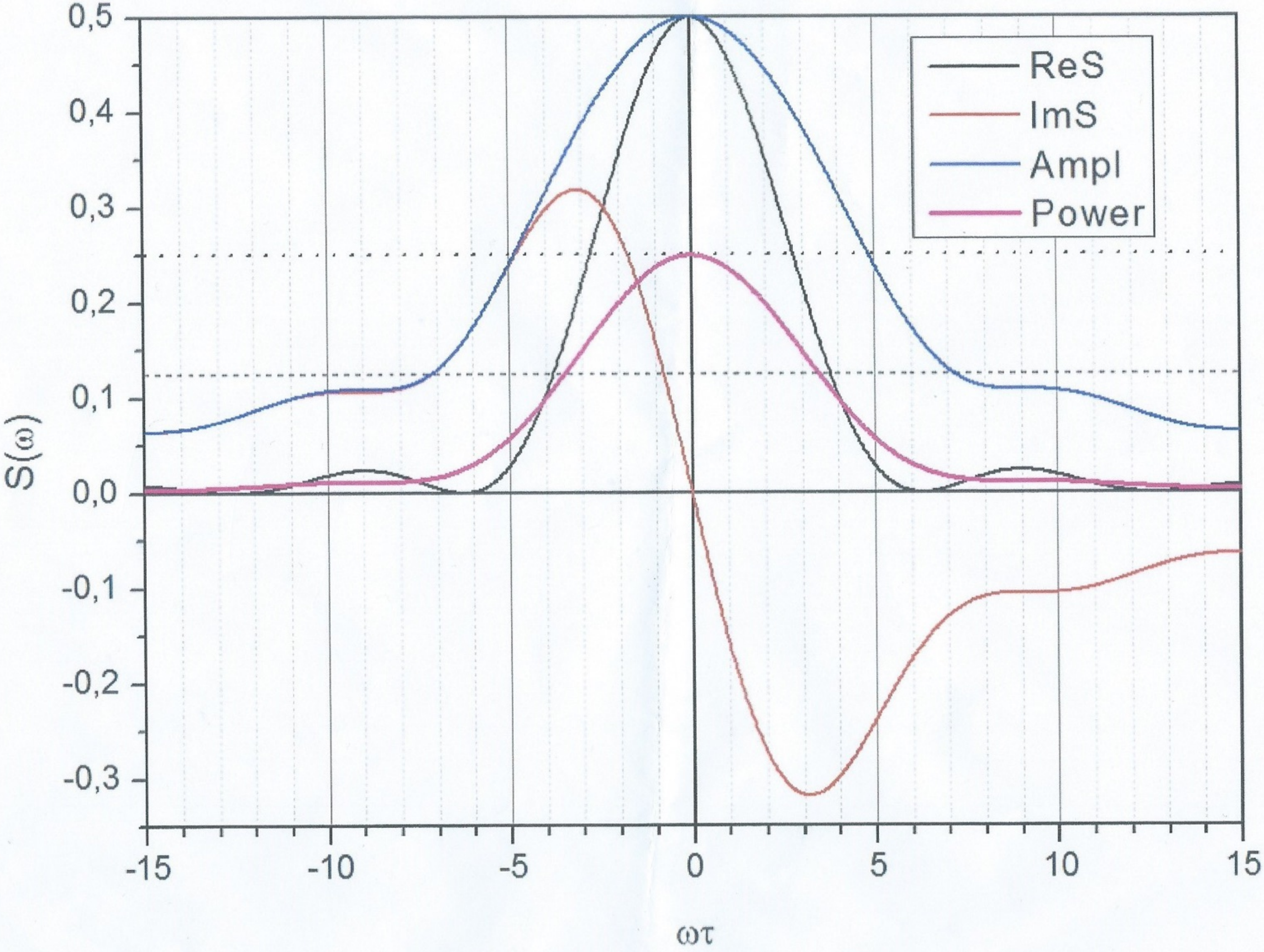


$$a_n = \frac{1}{\tau}\int_0^{\tau} f(t)\, e^{-in\omega_0 t}\, dt$$

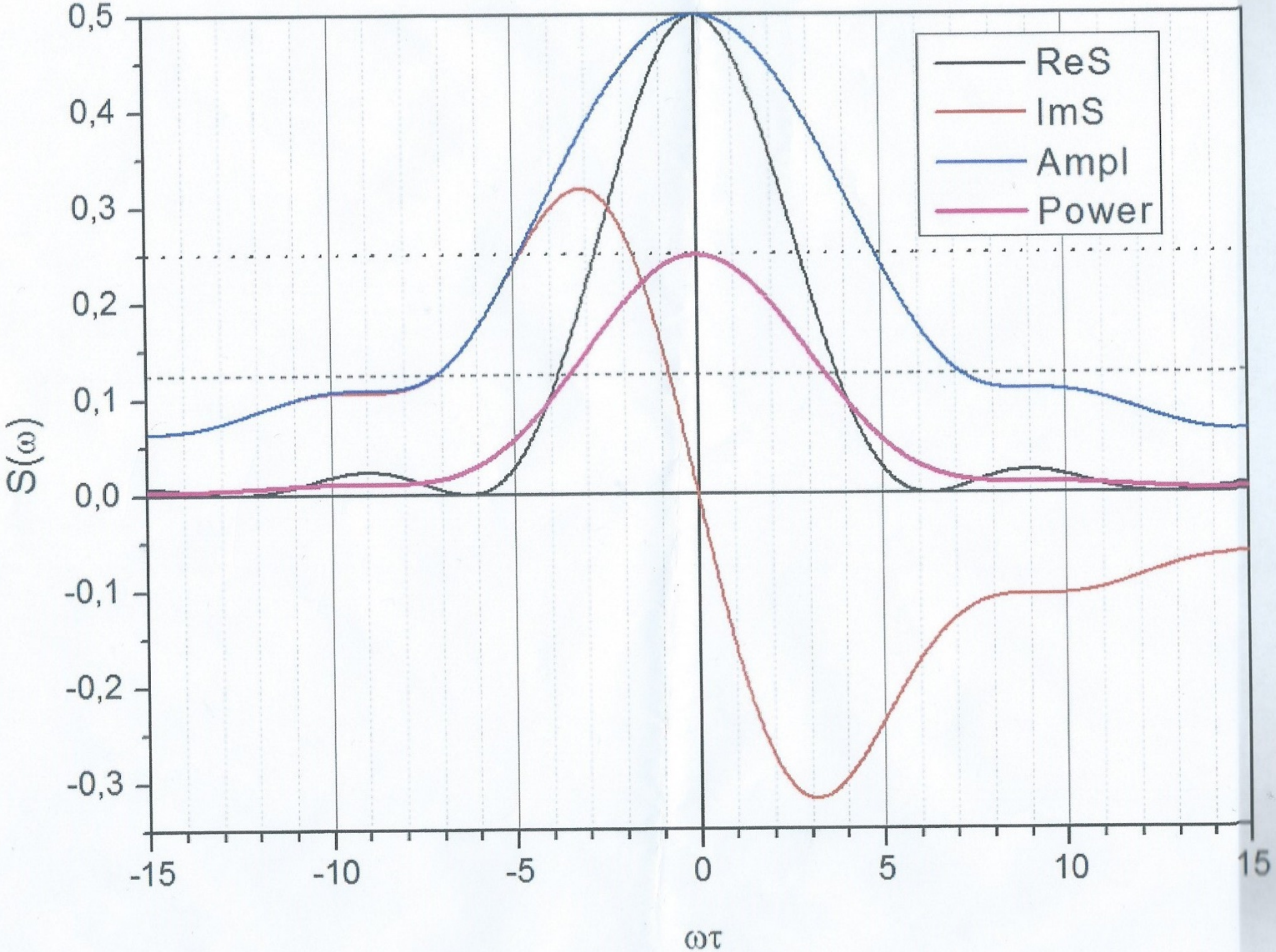


$f_0 = \frac{1}{\tau} = \begin{cases} 1\text{кГц} & 1\text{мс} \\ 1\text{МГц} & 1\text{мкс} \end{cases}$


Phase
Phase/π
ωτ
0,5
0,4
0,3
0,2
0,1
0,0
-0,1
-0,2
-0,3
-0,4
-0,5
-15
-10
-5
0
5
10
15



ReS
ImS
Ampl
Power
S(ω)
ωτ
0,5
0,4
0,3
0,2
0,1
0,0
-0,1
-0,2
-0,3
-15
-10
-5
0
5
10
15



ReS
ImS
Ampl
Power
S(ω)
ωτ
0,5
0,4
0,3
0,2
0,1
0,0
-0,1
-0,2
-0,3
-15
-10
-5
0
5
10
15